Bangs Laboratories, Inc.
BEADS ABOVE THE REST

Equations

TechNote 206

Values obtained through use of the following equations should be considered estimates, as the equations are based upon a number of theoretical assumptions. Values that are determined empirically or through use of analytical techniques are expected to differ to an extent. **Many values (such as % solids, diameter, density & surface area) are provided on the Certificate of Analysis and order paperwork that accompanies microsphere shipments.**

Solids Content

$$S = \frac{m \bullet 100}{V_S} \quad \text{OR} \quad m = \frac{S \bullet V_S}{100}$$

S	=	weight/volume % solids *(e.g., S = 10 for 5mL suspension, 0.5g microspheres)*
V_S	=	volume of suspension (mL)
m	=	mass of microspheres (g)

% Coefficient of Variation (Size Distribution of the Microsphere Population)

$$\% \, CV = \frac{SD}{d} \times 100$$

% CV	=	% coefficient of variation (size distribution of the microsphere population)
SD	=	standard deviation (µm) *(Note: Standard deviation is not provided for all products.)*
d	=	mean diameter (µm)

Microspheres/Gram

$$N = \frac{6 \times 10^{12}}{\pi \bullet \rho_S \bullet d^3}$$

N	=	# microspheres/gram for dry powders
ρ_S	=	density of solid sphere (g/cm^3)
d	=	mean diameter (µm)

Microspheres/mL

$$N = \frac{6 \times 10^{10} \bullet S \bullet \rho_L}{\pi \bullet \rho_S \bullet d^3}$$

N	=	# microspheres/mL for suspensions in water
S	=	weight % solids (for 10% solids suspension, S=10)
ρL	=	density of microsphere suspension (g/mL)
ρ_L	=	$100 \bullet \rho_S / [S\,(1-\rho_S) + (100 \bullet \rho_S)]$
ρ_S	=	density of solid sphere (g/cm^3)
d	=	mean diameter (µm)

Surface Area / Gram

$$AG = \frac{6 \times 10^{12}}{\rho_S \bullet d}$$

AG	=	surface area/gram for dry powders (µm^2/g)
ρ_S	=	density of solid sphere (g/cm^3)
d	=	mean diameter (µm)

Surface Area / mL

$$A = \frac{6 \times 10^{10} \bullet S \bullet \rho_L}{\rho_S \bullet d}$$

A	=	surface area / mL for suspensions in water (µm^2/g)
S	=	weight % solids (for 10% solids solution, S=10)
ρ_L	=	density of microsphere suspension (g/mL)
ρ_L	=	$100 \bullet \rho_S / [S\,(1-\rho_S) + (100 \bullet \rho_S)]$
ρ_S	=	density of solid sphere (g/cm^3)
d	=	mean diameter (µm)

Parking Area (Surface Charge Density)

$$P = \frac{1}{1.004 \bullet D_C \bullet \rho_S \bullet d}$$

P	=	parking area (Å^2/charge group)
DC	=	surface charge density or titration value (meq/g) (provided in µeq/g on COA)
ρ_S	=	density of solid sphere (g/cm^3)
d	=	mean diameter (µm)
		Notes: Surface titer / parking area are not provided for all products.

Charge Groups / Microsphere

$$C_M = \frac{\pi \bullet d^2 \bullet 10^8}{P}$$

C_M	=	# charge groups / microsphere
d	=	mean diameter (µm)
P	=	parking area (Å^2)

Settling Velocity (Specify Medium)

$$v = \frac{1}{18} d2 (\rho_S - \rho_0) \frac{10^{-6}}{h} G$$

v	=	settling velocity (cm / sec)
d	=	mean diameter (µm)
ρ_S	=	density of solid sphere (g/cm^3)
ρ_0	=	medium density (g/cm^3)
h	=	medium viscosity (poise = g/cm • sec)
G	=	gravity (980 cm/sec^2)

Settling Velocity (In Water)

$$V_m = 5.448 \times 10^{-5} \bullet (\rho_S - 1) \bullet d^2$$

Vm	=	maximum settling velocity (cm/sec) for a single microsphere settling in water at room temperature under the influence of normal gravitational force (1G)
ρ_S	=	density of solid sphere (g/cm^3)
d	=	mean diameter (µm)

Settling Velocity (In the Centrifuge)

$$V_h^{5\%} = \frac{2}{3} \bullet V_m \bullet G$$

$V_h^{5\%}$	=	true settling velocity or hindered velocity (cm/sec) for a 5% w/w suspension of microspheres settling in water at room temperature
V_m	=	maximum settling velocity
G	=	multiples of earth gravitation constant, G forces

Settling Time

$$t = \frac{h}{V_h^{5\%}} \quad \text{OR} \quad \frac{h}{V_{ch}^{5\%}}$$

t	=	settling time (sec)
h	=	distance from the top of the liquid to the bottom layer of settled solids (cm)
V_h^{5}%	=	true settling velocity or hindered velocity (cm/sec) for a 5% w/w suspension of microspheres settling in water at room temperature under the influence of normal gravitational force (1G)
$V_{ch}^{5\%}$	=	hindered settling velocity in the centrifuge (cm/sec) for a 5% w/w suspension of microspheres settling in water at room temperature

Surface Saturation (Protein)

$$S = \frac{6}{\rho_S d} \bullet C$$

S	=	amount of representative protein required to achieve surface saturation (mg protein/g microspheres)
ρ_S	=	density of solid sphere (g/cm^3)
d	=	mean diameter (µm)
C	=	capacity of microsphere surface for a given protein (mg protein/m2 of sphere surface)

Notes: C ~ 3 mg/m^2 for BSA [MW 65kD], C ~ 2.5 mg/m^2 for bovine IgG [MW 150kd].[1] See Tech Note 204, *Adsorption to Microspheres*, and TechNote 205, *Covalent Coupling*, for more detailed information.

SAMPLE VALUES

Diameter (Microns)	Beads per gram	Beads per mL	Surface Area ($\mu m^2/g$)	Surface Area ($\mu m^2/mL$)	Settling Velocity (cm/sec)
0.052	$1.3x10^{16}$	$1.3x10^{15}$	$1.1x10^{14}$	$1.1x10^{13}$	$7.4x10^{-9}$
0.100	$1.8x10^{15}$	$1.8x10^{14}$	$5.7x10^{13}$	$5.7x10^{12}$	$2.7x10^{-8}$
0.500	$1.5x10^{13}$	$1.5x10^{12}$	$1.1x10^{13}$	$1.1x10^{12}$	$6.8x10^{-7}$
1.000	$1.8x10^{12}$	$1.8x10^{11}$	$5.7x10^{12}$	$5.7x10^{11}$	$2.7x10^{-6}$
2.500	$1.2x10^{11}$	$1.1x10^{10}$	$2.3x10^{12}$	$2.3x10^{11}$	$1.7x10^{-5}$
10.00	$1.8x10^{9}$	$1.8x10^{8}$	$5.7x10^{11}$	$5.7x10^{10}$	$2.7x10^{-4}$
25.00	$1.2x10^{8}$	$1.2x10^{7}$	$2.3x10^{11}$	$2.3x10^{10}$	$2.0x10^{-3}$
108.0	$1.4x10^{6}$	---	$5.3x10^{10}$	---	---
500.0	$1.4x10^{4}$	---	$1.1x10^{10}$	---	---
Notes: Calculations for 0.052-25.0µm are based on a suspension of polystyrene microspheres (density = 1.05 g/cm^3) at 10% solids (w/v).108 and 500µm diameter microsphere calculations compositions are based on compositions of poly(styrene/2% divinylbenzene), density = 1.06 g/cm^3, and the calculations are based on dry presentation.					

REFERENCES

1. Cantarero, L.A., J.E. Butler, J.W. Osborne. 1980. *The adsorptive characteristics of protein for polystyrene and their significance for solid-phase immunoassays.* Analytical Biochemistry, 105: 375-382.
2. Bangs, L.B. 1987. *Uniform latex particles*. Indianapolis: Seragen Diagnostics, Inc.